ANALYSIS OF SYMMETRIC-VARACTOR FREQUENCY TRIPLERS

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ABSTRACT

A simple analytical model for varactors that have a symmetric capacitance-voltage characteristic is introduced. This empirical but general model can be used to represent the reactance nonlinearity of quantum-barrier varactors, symmetric high-electron-mobility varactors and back-to-back BNN$^+$ diodes. Using this model, a generalized large-signal analysis of symmetric-varactor triplers is developed. The analysis yields an expression for the maximum efficiency. It is further shown that a series-resonant circuit including a symmetric varactor can be described by Duffing's equation. The solution of this equation predicts hysteresis effects.

INTRODUCTION

New classes of varactors having symmetrical capacitance-voltage [$C(v)$] characteristics are under development [1,2,3]. They include the symmetrical quantum-barrier varactor (S-QBV), the symmetric high-electron-mobility varactor (S-HEMV), and the back-to-back *barrier-n-n*$^+$ (bb-BNN$^+$) varactor. Because they display their maximum nonlinearity near zero bias, these devices are promising as efficient triplers at millimeter-wave frequencies where source power is at a premium. The prediction of the maximum possible optimized efficiency of of such triplers has not previously been reported, primarily because of the lack of a simple analytical model. This paper presents such a model, and uses it to predict the maximum performance of symmetric-varactor triplers. It is further shown that when the new model is embedded in a simplified series-resonant tripler circuit, the behavior is described by the classical Duffing equation [4], and that the solution shows jump phenomena, typical of nonlinear-reactance circuits.

THE DEVICE MODEL

The device model consists of a nonlinear capacitor in series with a loss resistance R_s representing the combined effects of

(a) The skin-effect resistance
(b) The contact resistance
(c) The resistance of the undepleted epilayer. We propose the following general empirical charge-voltage model for all the symmetric $C(v)$ varactors:

$$\alpha v = \frac{q}{q_0} + \beta \left(\frac{q}{q_0}\right)^3 \quad (1)$$

where v is the potential across the varactor (excluding R_s) and q is the charge associated with the varactor depletion region. In terms of normalized variables, (1) becomes simply

$$y = z + \beta z^3 . \quad (2)$$

where $y = \alpha v$ and $z = q/q_0$. The parameters α, β and q_0 can be adjusted to fit the q-v relation (1) to the measured characteristics of an arbitrary symmetrical $C(v)$ varactor. From (2) one can find the following explicit $C(v)$ relation to be fitted to measured data:

$$y = \alpha v = \pm \psi^{1/2}[1 + \beta \psi] \quad (3)$$

where

$$\psi = \frac{1}{3\beta}\left(\frac{C(0)}{C(v)} - 1\right) . \quad (4)$$

Figure 1 shows the $C(v)$ of a S-QBV calculated from the approximate expression [5]

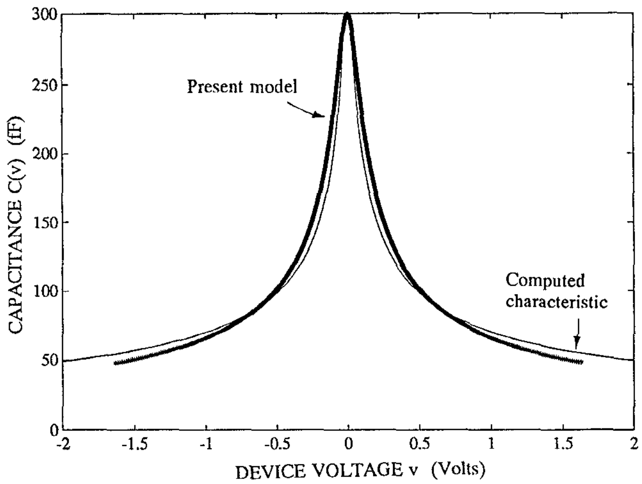

Figure 1. The theoretical symmetric quantum barrier $C(v)$ characteristic compared with the proposed model. The fitting parameters are set to α = 2.2 and β = 3.2.

$$C(v) = \varepsilon A \Big/ \left(w + \sqrt{2\varepsilon\,|v_d|\,/(eN_D)}\right) \tag{5}$$

where ε is the dielectric constant, A is the device area, w is the effective barrier thickness, e is the magnitude of the electron charge, N_D is the doping density of the depletion region, and v_d is the portion of the applied v that appears across the depletion region. The rest of v is divided between the QBV accumulation and barrier regions. In the case illustrated A = 70 $(\mu m)^2$, $C_{max} = C(0)$ = 300 fF, and $C_{min} = C(v = 2\text{ V})$ = 50 fF. The figure also shows the $C(v)$ according to the present model, with the fitting parameters set to $\alpha = 2.2$ and $\beta = 3.2$. This device is ideally suited for application in a low-power frequency tripler. Most of the device nonlinearity is concentrated in the vicinity of zero bias. For $|v|$ greater than ~ 2.0 V the reactance nonlinearity is slight, and the conduction current of the device starts to increase, lowering its Q.

ANALYSIS OF FREQUENCY TRIPLERS

Triplers using S-QBVs and bbBNN$^+$ varactors have been reported [1,2]. Previous analyses were numerical and employed the harmonic-balance (HB) technique. In S-QBV and S-HEMV devices the very strong $C(v)$ nonlinearity around zero bias can lead to numerical instability in HB simulations. Furthermore, HB simulations are computer intensive and do not easily offer insights into the performance of the device in a tripler circuit.

A simple large-signal analysis of classical abrupt-junction pn-junction varactors in terms of a q-v model has been given by Tang [6]. The analysis given here follows Tang's approach. The device equivalent circuit consists of a series combination of $C(v)$ and R_s. The tripler equivalent circuit in Fig. 2 consists of the symmetrical varactor model placed

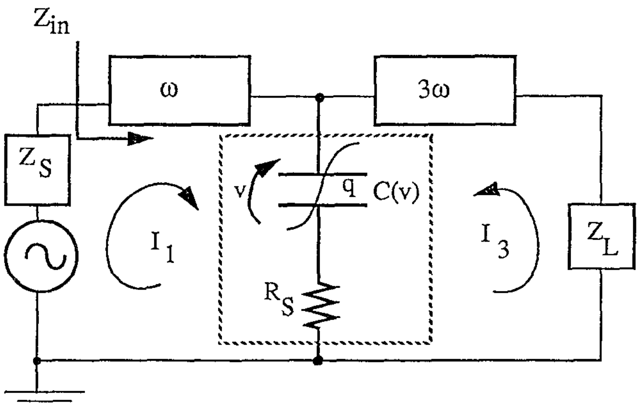

Figure 2. Tripler circuit model for analysis.

between ideal band-pass filters at angular frequencies ω and 3ω. The source and load impedances are Z_S and Z_L respectively. The total current flowing through the varactor diode is

$$i = I_1\cos(\omega t) + I_3\cos(3\omega t) \tag{6}$$

while the normalized charge associated with its depletion layer is

$$z = \frac{q}{q_0} = \frac{I_1}{\omega q_0}\sin(\omega t) + \frac{I_3}{3\omega q_0}\sin(3\omega t + \theta)\,. \tag{7}$$

In the ideal case of perfect q-v antisymmetry described by (1), the average charge is zero. The input power at ω is assumed to be sufficiently low that the diode is not driven to its breakdown voltage V_B. Substituting (7) into (2) the normalized voltage y across the lossless ideal $C(v)$ element can be found. This voltage has third, fifth, seventh and ninth harmonics (but no higher-order terms):

$$y = \alpha v = Y_1\cos(\tau - \zeta_1) + Y_3\cos(3\tau - \zeta_3) + \ldots \tag{8}$$

where $\tau = \omega t$, and Y_n and ζ_n are amplitudes and phases to be determined. Imposing the circuit conditions one finds the following equations:

$$Y_1\cos\zeta_1 = -\frac{3}{4}\beta Z_1^2 Z_3\sin\theta \tag{9}$$

$$Y_1\sin\zeta_1 = Z_1\left[1 + \frac{3}{2}\beta\left(\frac{1}{2}Z_1^2 + Z_3^2\right)\right] - \frac{3}{4}\beta Z_1^2 Z_3\cos\theta \tag{10}$$

$$Y_3\cos\zeta_3 = Z_3\left[1 + \frac{3}{2}\beta\left(Z_1^2 + \frac{1}{2}Z_3^2\right)\right]\sin\theta \tag{11}$$

$$Y_3\sin\zeta_3 = Z_3\left[1 + \frac{3}{2}\beta\left(Z_1^2 + \frac{1}{2}Z_3^2\right)\right]\cos\theta - \frac{1}{4}\beta Z_1^3 \tag{12}$$

where Z_1 and Z_3 are the amplitudes of the normalized fundamental and third-harmonic charge components.

Defining the constraints to be (a) fixed input power P supplied to the nonlinear $C(v)$ element (i.e. not including R_s), and (b) minimum dissipation with respect to I_1, the optimization condition is found to be

$$I_1 = (12)^{1/8}\left(\frac{P}{M\sin\theta}\right)^{1/4} = \sqrt{3}\,I_3 \tag{13}$$

where

$$M = \frac{\beta}{4\alpha(\omega q_0)^3}\,. \tag{14}$$

The tripler efficiency is

$$\eta = \frac{\textit{power output}}{\textit{power input}} = \frac{P - \frac{1}{2}I_3^2R_s}{P + \frac{1}{2}I_1^2R_s}\,. \tag{15}$$

Substituting the optimization condition (13) into (15), the efficiency is

$$\eta = \frac{\sqrt{6\sqrt{3}}(PM\sin\theta)^{1/2} - R_s}{\sqrt{6\sqrt{3}}(PM\sin\theta)^{1/2} + 3R_s} . \qquad (16)$$

The maximum efficiency occurs when the phase angle is $\theta = 90^0$. This is also the condition for minimum dissipation.

The input and output impedances are found to be

$$Z_{in} = \left(\frac{V_1}{I_1}\cos\zeta_1 + R_s\right) + j\left(\frac{V_1}{I_1}\sin\zeta_1\right) \qquad (17)$$

and

$$Z_L = \left[\frac{-V_3}{I_3}\cos(\zeta_3 - \theta) - R_s\right] + j\left[\frac{-V_3}{I_3}\sin(\zeta_3 - \theta)\right] . \qquad (18)$$

It is interesting to note that the real parts of the input and load resistances are related by

$$\frac{R_{in}}{R_L} = \frac{1}{3\eta} . \qquad (19)$$

The maximum efficiency of a tripler using a symmetric quantum-barrier varactor with the $C(v)$ characteristic denoted "present model" in Fig. 1 is plotted in Fig. 3. The large-signal cutoff frequency can be found from the Penfield and Rafuse

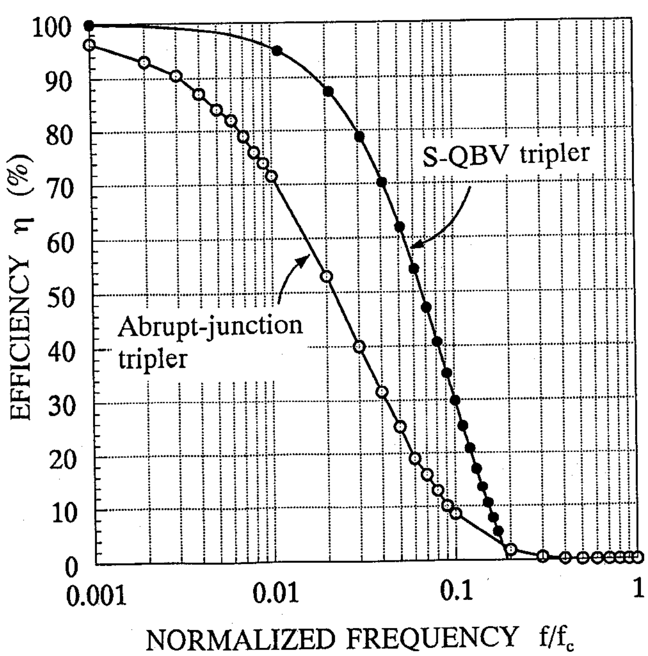

Figure 3. Comparison of the maximum efficiency of a S-QBV tripler (with α = 2.2, β = 3.2) with that of a conventional abrupt-junction tripler using a short-circuit idler.

definition [7] $f_c = 1/(2\pi R_s C_{min})$. Using C_{min} = 50 fF and R_s = 10 Ω one obtains f_c = 318.8 GHz. In Fig. 3 the efficiency of the S-QBV tripler is compared with that of a classical abrupt-junction tripler [7] using a short-circuit idler at the second harmonic.

Other relevant design information can be calculated from the above analysis, and is listed in Table I for for a tripler from 30 to 90 GHz.

Table I: Data for an S-QBV tripler from 30 to 90 GHz with α = 2.2 and β = 3.2:

P_{in}	η	P_{out}	R_{in}	X_{in}	R_L	X_L
8.0 mW	33.13 %	2.65 mW	19.93 Ω	-73.15 Ω	19.84 Ω	40.52 Ω

Since most of the device nonlinearity is concentrated around zero bias voltage, only a small input RF power is needed to modulate (pump) that nonlinearity. The fundamental-frequency voltage across the $C(v)$ element found to be 2.1 V. The calculated efficiency somewhat overestimates reality because the present theory does not include the effect of Γ-X transfer current in the QBV [5].

DUFFING'S EQUATION

Consider a varactor device defined by the charge-voltage representation (1) to be embedded in the series-resonant circuit of Fig. 4. Here R can be considered a

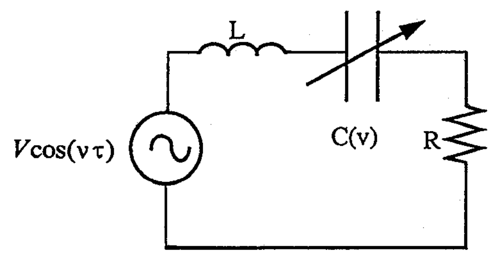

Figure 4. A symmetric varactor embedded in a series-resonant circuit.

combination of R_L and R_s. It can be shown that the differential equation describing the circuit is

$$\overset{\circ\circ}{z} + \zeta\overset{\circ}{z} + z + \beta z^3 = X\cos(\nu\tau) \qquad (20)$$

where $\zeta = R/(\omega_0 L)$ is a damping term, $\omega_0 = 1/\sqrt{[LC(0)]}$, $\tau = \omega_0 t$, $X = \alpha V$ is the amplitude of the forcing function, $^\circ \equiv d/d\tau$, and $\nu = \omega/\omega_0$. This differential equation is the well-known Duffing's equation [4], approximate solutions of which have been described in the mathematical literature [7,8] for small values of the parameters β, ζ, and X. Assuming a normalized-charge solution

$$z \approx Z_3\cos(3\nu\tau - \gamma) + Z_1\cos(\nu\tau) \qquad (21)$$

and using the method of multiple scales [8] a solution can be found for the normalized third-harmonic charge amplitude Z_3:

$$\left[\frac{\zeta}{2}\right]^2 Z_3^2 + \left[\left\{3\nu - 1 - 3\beta\left[\frac{X}{2(1-\nu^2)}\right]^2\right\}Z_3 - \frac{3}{8}\beta Z_3^3\right]^2 = \beta^2\left[\frac{X}{2(1-\nu^2)}\right]^6 . \qquad (22)$$

The frequency-response diagram of Fig. 5 shows Z_3 as a function of the normalized input frequency ν. Examination of this curve shows the onset of a jump phenomenon, a common occurence with nonlinear-reactance circuits.

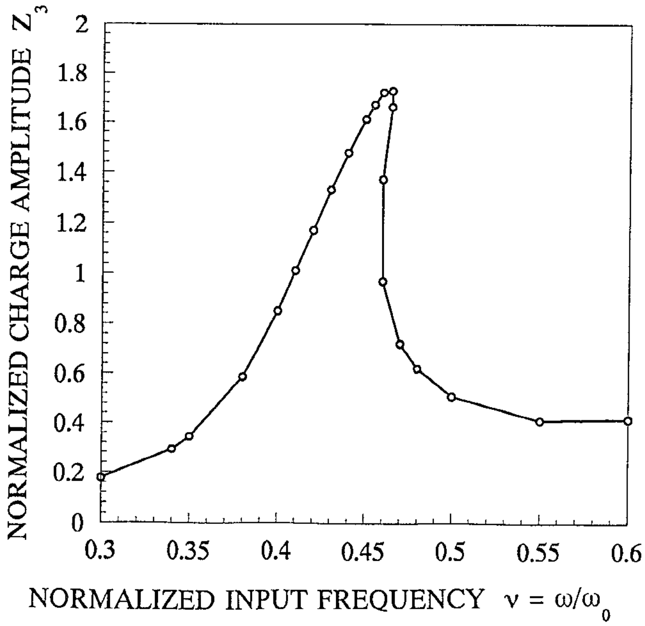

Figure 5. Amplitude of the normalized third-harmonic varactor charge as a function of the input frequency, as given by the solution of Duffing's equation for $\beta = 0.1$, $\zeta = 0.1$, $X = 1.5$. Note that the frequency at the peak is far from $\nu = 1/3$ because of the rapid decrease of average capacitance under pumped conditions. Also note the onset of hysteresis.

CONCLUSION

We have proposed a simple mathematical nonlinear model that can be used to represent a variety of different types of varactor exhibiting symmetric $C(v)$ characteristics.

We have employed this model to carry out a closed-form large-signal analysis of a frequency tripler circuit. The results show that the optimized efficiencies at low input power levels are greater than those possible for conventional varactors having similar cutoff frequencies and breakdown voltages. The new model, suitably augmented with parasitic elements to more closely represent real symmetric varactors, can be used in computer-aided design programs.

We have further demonstrated that when the new model is embedded in a prototype series-resonant tripler configuration, the circuit obeys Duffing's equation, and that the solution shows jump phenomena, typical of nonlinear-reactance circuits.

ACKNOWLEDGEMENT

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